Collision Rate of Small Particles in a Homogeneous and Isotropic Turbulence

A theoretical equation is derived for the collision rate of aerosol particles in a homogeneous and isotropic turbulent system. This equation takes into account the relative velocity between fluid and particles. The calculated results indicate that the relative velocity between fluid and particles is the main factor in the turbulent coagulation (agglomeration, coalescence) of unequally sized particles in an air flow. This hold true, even when the particle sizes are less than 1 micron. For particles of equal radii the coagulation coefficient reaches its minimum value, because the effect of motion relative to the fluid now becomes zero and only the spatial variation of turbulent motion remains to cause collisions between the particles. For particles following a fluid motion completely, as in a water stream, the equation for the collision rate reduces to the Saffman and Turner equation.

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SCOPE

Turbulence causes collisions between neighboring particles by increasing their random motion. These collisions then result in agglomeration of particles or coalescence of droplets (refered to subsequently as "coagulation of particles"). Since most flows in which particles are suspended are turbulent, it is important to understand this process and to determine the effect that it has on particle coagulation and on the change with time of particle concentration and size distribution. An approach to this is to first determine the collision rate. Then the coagulation process can be determined by the numerical calculation of the population balance equation.

Saffman and Turner (1956) derived the collision rate equation in an isotropic turbulent flow by considering the spatial variations of turbulent velocity. Their result indicates that the collision rate increases proportionally to the square root of the viscous dissipation rate of the fluid. Levich (1962) and Beal (1972) obtained a similar result of Saffman and Turner, considering the particle collision as the deposition of one particle onto another particle due to the diffusion process. However, these studies did not have completely taken into account the relative velocity between fluid and particle. Even a slight relative velocity between fluid and particles affects the collision rate, and this should not be neglected even for particles under 1 μm in size.

The objective of this study is to describe theoretically the mechanism of the coagulation of small particles in a turbulent flow. To do this a theoretical equation for the collision rate of small particles in a homogeneous and isotropic turbulence is derived as follows. First, the variance of particle relative velocities is expressed as in terms of the turbulent intensity of each particle, the variance of the particle velocity gradient, and their correlation. These turbulent characteristics of the particles are then converted into those of the fluid by means of the equation of particle motion and by the empirical formula for the Lagrangian fluid velocity correlation. Finally, the equation of collision rate is obtained as a function of particle radii, particle relaxation times, fluid turbulent intensities, dissipation rate, and fluid integral time scales.

CONCLUSIONS AND SIGNIFICANCE

An equation for the collision rate of particles in turbulent flow is derived theoretically, taking into account the relative velocity between fluid and particle. The results obtained by calculating the coagulation coefficient indicate that the slight relative velocity between the different sized particles, caused by their different inertias, is the predominate factor in the coagulation process and holds true even when the particle diameters are less than 1 μm. For example, if the radius of one particle is 0.5 μm and the other is less than 0.3 μm, their collision rate due to their relative velocities is about four times larger than that due just to the spatial distribution of turbulence. For particles with equal radii, the coagulation coefficient is at its minimum value, because their motions relative to air are zero and the only factor causing collisions is the spatial distribution of turbulence.

For particles much smaller than the small eddies of the usual isotropic turbulence, i.e., less than about 50 micron, the collision rates are calculated in terms of the particle relaxation time, the integral time scale, the dissipation rate of turbulence, and the fluid turbulent intensity.

The most important factors affecting turbulent coagulation are the dissipation rate of turbulence and the particle relaxation time (the particle inertia). The collision rates due to the motion relative to the surrounding fluid and to the spatial variation of the turbulence increase with an increase in the dissipation rate of turbulence.

This theoretical equation of particle collision rate will permit a better prediction of particle coagulation in turbulent flow.

PREVIOUS WORK

The problem of the collision of particles suspended in a turbulent flow medium has been studied by earlier workers. They have been valuable to us as background and a starting point for our present study. Smoluchowski (1917) derived an equation for the collision rate in a simple shear flow. Camp and Stein (1943) applied this to a turbulent coagulation process and obtained the following equation.

$$N_{12} = \frac{4}{3} n_1 n_2 (r_{p1} + r_{p2})^3 (\epsilon/\nu)^{1/2} \tag{1}$$

Saffman and Turner (1956) derived a similar equation considering the effect of the spatial variations of turbulent motion. Levich (1962) and Beal (1972) considered the turbulent collision process as the deposition of particles onto a sink particle and derived a similar equation. However, in all of the above, the effect of the relative velocities between the suspending fluid and the particles was not completely considered. Therefore, this equation does not express the situation over a sufficiently wide range of conditions.

East and Marshall (1954) considered the role that turbulent motion could play in precipitation and concluded that its effect was equivalent to that of an increased gravitational field. However, they neglected consideration of the spatial variations of turbulent motion which are important for the collision of equally sized particles.

In a further derivation, Saffman and Turner (1956) took account also of the relative velocity between suspending fluid and particles and derived an equation, which can be written as

$$N_{12} = (8\pi)^{1/2}(r_{p_1} + r_{p_2})^2 n_1 n_2 \left\{\left(1 - \frac{\rho_f}{\rho_p}\right)^2 (\tau_{p_1} - \tau_{p_2})^2 \left[\left(\frac{Dv_f}{Dt}\right)^2 + \frac{1}{3}g^2\right] + \frac{1}{9}(r_{p_1} + r_{p_2})^2 \epsilon/\nu\right\}^{1/2} \quad (2)$$

where D/Dt indicates the substantial derivative. However, there appear to be two important factors which limit the use of this equation.

1. The collision rate N_{12} increases in proportion to the relaxation time of particle τ_{pi} when $r_{p_1} \neq r_{p_2}$. This means that when τ_{pi} becomes large, N_{12} increases without limits. However, when τ_{pi} becomes very large, the particle does not follow the turbulent motion of the fluid. As a result, the collision rate due to turbulence decreases because the turbulent components of particle motion become smaller. This contradiction in Eq. 2 is attributable to the fact that the acceleration term in the equation of motion of particles has been neglected.

2. When $r_{p_1} = r_{p_2}$, N_{12} depends only on the characteristic value of ϵ/ν for the fluid and on the particle size. Since a particle cannot follow the fluid motion completely, the effect of velocity lag should be taken into account, even in the case of collision of particles of the same size. This contradiction is also attributable to neglecting the acceleration term in the equation of motion of particles.

In view of these contradictions it can be seen that the definitive theoretical equation for estimating the collision rate of particles in turbulent flow has not yet been derived. In the work that follows we are attempting to derive an equation which still more closely approximates the real situation.

THEORY

The calculations in this study were performed making the following assumptions

1) The turbulence characteristics are not affected by the presence of particles or droplets.
2) The turbulence is isotropic and homogeneous.
3) The effect of gravity and Brownian movement on the collision rate is negligible.
4) The drag on each particle is determined by Stokes' law.

These are all reasonable assumptions and do not impose serious limitations on use of the final derived equation. The first can be reasonably applied when the mixing ratio on a weight basis is less than 0.1. The second is essentially applicable to cases such as the turbulent flow fields at the downward side of a grid, a stirred tank with a high rotational velocity, or the main flow region in a pipe. It can also be applied, without too much error in the final results, to a large number of other situations where deviations from isotrophy and homogeneity are not too large.

When the collision rates from gravity or Brownian movement are compared with those from turbulence, the former are usually smaller by at least an order of magnitude and can therefore be ignored. Finally, the Reynolds number is less than 1, justifying the fourth assumption in a water stream. For air streams, Cunningham's slip correction to Stokes' law is used.

Collision Rate and Coagulation Coefficient

When suspended and uniformly distributed particles have velocities relative to each other, they collide and coagulate (agglomerate or coalesce). Accordingly, the most important physical variable is the collision rate, which is defined as the number of collisions per unit time and unit volume.

The probability distribution of relative velocities of two particles is a function of the separation of their centers. If the separation is small compared with the diameter of the turbulence eddies, the probability distribution can be derived from the equation of particle motion and the statistical properties of turbulence. A collision occurs when the separation of the particle centers is equal to $r_{p_1} + r_{p_2}$. Therefore, the collision volume per unit time V_{12} (the volume in which all of the included particles collide with the other particle at rest per unit time) is equal to $\pi(r_{p_1} + r_{p_2})^2|w|$, where w is the relative velocity vector of two colliding particles. However, as two particles approach each other the stream lines of the fluid are distorted by the static layers next to each particle and the particles tend to be pushed away from each other, and a collision does not always occur. Therefore, collision efficiency should be taken into account. The effective collision volume V_{12} is then

$$V_{12} = \pi\eta(r_{p_1} + r_{p_2})^2|w|. \quad (3)$$

The number concentrations of the particles of diameter r_{p_1} and r_{p_2} are denoted by n_1 and n_2, and the collision rate N_{12} is derived by multiplying the product of V_{12} and n_1n_2 by the value of the integration of $p(w)$ over the whole region of w, to account for the fact that all the particles contained in the volume V_{12} collide per unit time.

$$N_{12} = \pi\eta(r_{p_1} + r_{p_2})^2 n_1 n_2 \iiint_{-\infty}^{+\infty} |w| p(w) dw. \quad (4)$$

A necessary condition here is that the mean velocity of colliding particles is statistically independent of their relative velocity; otherwise, the particle concentration has to be related to the relative velocity.

In a turbulent system the mean velocity is controlled by the large eddies and the relative velocity by the small eddies. The large and small eddies are statistically independent when the turbulent Reynolds number is large, which is the case with the systems which we have under study. Therefore, it is allowable to multiply by the number concentration of the particles in the above-mentioned manner.

The probability distribution of the relative velocities among particles, $P(w)$, is considered to be expressed as a Gaussian form in a homogeneous and isotropic turbulent system.

$$P(w) = \left(\frac{3}{2\pi\overline{w^2}}\right)^{3/2} \exp\left(-\frac{3w^2}{2\overline{w^2}}\right). \quad (5)$$

Substitution of Eq. 4 into Eq. 5 yields

$$N_{12} = \pi\eta(r_{p_1} + r_{p_2})^2 n_1 n_2 \iiint_{-\infty}^{+\infty} |w| \cdot \exp\left[-\frac{3(w_x^2 + w_y^2 + w_z^2)}{2\overline{w^2}}\right]\left(\frac{3}{2\pi\overline{w^2}}\right)^{3/2} \cdot dw_x dw_y dw_z = 2\left(\frac{2\pi}{3}\right)^{1/2} \eta(r_{p_1} + r_{p_2})^2 n_1 n_2 \sqrt{\overline{w^2}}. \quad (6)$$

using the transformations $w_x = |w|\cos\theta\cos\psi$, $w_y = |w|\cos\theta\sin\psi$, and, $w_z = |w|\sin\theta$ to perform the integration. Equation 6 is similar to one obtained by Saffman and Turner (1956). When the value of the root mean square of the particle relative velocities is known, N_{12} is calculated from Eq. 6. The variance of particle relative velocities contains the effects of the spatial variations of fluid turbulent velocity and the particle motion relative to the fluid.

Variance of Particle Relative Velocity

As shown in Figure 1, $v_{p_{11}}$ and $v_{p_{22}}$ denote the velocity vectors of two particles with their centers at position 1 and 2, while $v_{p_{10}}$ and $v_{p_{20}}$ denote the hypothetical velocity vectors of the particles with

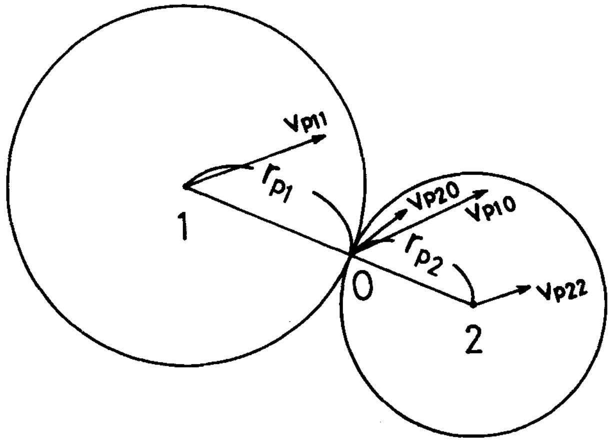

Figure 1. Collision model.

their centers at the contact point (position 0). To clarify the effects of the spatial variations of fluid turbulent velocity and of the particle motion relative to the fluid, $v_{p_{11}}$ and $v_{p_{22}}$ should be expressed in terms of $v_{p_{10}}$ and $v_{p_{20}}$. Since r_{p_1} and r_{p_2} are very small, the gradients of the velocities over the distance from the position 0 to the position 1 or 2 can be considered as constant. As the position 0 lies between the two positions 1 and 2, $v_{p_{11}}$ and $v_{p_{22}}$ are

$$v_{p_{11}} = v_{p_{10}} \mp r_{p_1} \cdot \frac{\partial v_{p_{10}}}{\partial s}, \qquad v_{p_{22}} = v_{p_{20}} \pm r_{p_2} \cdot \frac{\partial v_{p_{20}}}{\partial s}. \quad (7)$$

The variance of particle relative velocities is then

$$\overline{w^2} = \overline{(v_{p_{22}} - v_{p_{11}})^2} = \overline{v_{p_{22}}^2} - \overline{2 v_{p_{22}} v_{p_{11}}} + \overline{v_{p_{11}}^2}. \quad (8)$$

Substitution of Eq. 7 into Eq. 8 yields

$$\overline{w^2} = \overline{(v_{p_{20}} - v_{p_{10}})^2} + \overline{\left(r_{p_2} \cdot \frac{\partial v_{p_{20}}}{\partial s}\right)^2} + 2\,\overline{\left(r_{p_1} \cdot \frac{\partial v_{p_{10}}}{\partial s}\right)\left(r_{p_2} \cdot \frac{\partial v_{p_{20}}}{\partial s}\right)} + \overline{\left(r_{p_1} \cdot \frac{\partial v_{p_{10}}}{\partial s}\right)^2} \pm \overline{2(v_{p_{20}} - v_{p_{10}})\left(r_{p_1} \cdot \frac{\partial v_{p_{10}}}{\partial s} + r_{p_2} \cdot \frac{\partial v_{p_{20}}}{\partial s}\right)}. \quad (9)$$

The real variance is the arithmetic average of two cases expressed by the plus-minus sign in Eq. 9, so the following equation is obtained.

$$\overline{w^2} = \overline{(v_{p_{20}} - v_{p_{10}})^2} + \overline{\left(r_{p_2} \cdot \frac{\partial v_{p_{20}}}{\partial s} + r_{p_1} \cdot \frac{\partial v_{p_{10}}}{\partial s}\right)^2} = \overline{v_{p10}^2} + \overline{v_{p20}^2} - \overline{2 v_{p_{10}} \cdot v_{p_{20}}} + r_{p_{1x}}^2 \overline{\left(\frac{\partial v_{p_{10}}}{\partial x}\right)^2} + r_{p_{1y}}^2 \overline{\left(\frac{\partial v_{p_{10}}}{\partial y}\right)^2} + r_{p_{1z}}^2 \overline{\left(\frac{\partial v_{p_{10}}}{\partial z}\right)^2} + r_{p_{2x}}^2 \overline{\left(\frac{\partial v_{p_{20}}}{\partial x}\right)^2} + r_{p_{2y}}^2 \overline{\left(\frac{\partial v_{p_{20}}}{\partial y}\right)^2} + r_{p_{2z}}^2 \overline{\left(\frac{\partial v_{p_{20}}}{\partial z}\right)^2} + 2 r_{p_{1x}} r_{p_{2x}} \overline{\frac{\partial v_{p_{10}}}{\partial x} \cdot \frac{\partial v_{p_{20}}}{\partial x}} + 2 r_{p_{1y}} r_{p_{2y}} \overline{\frac{\partial v_{p_{10}}}{\partial y} \cdot \frac{\partial v_{p_{20}}}{\partial y}} + 2 r_{p_{1z}} r_{p_{2z}} \overline{\frac{\partial v_{p_{10}}}{\partial z} \cdot \frac{\partial v_{p_{20}}}{\partial z}}. \quad (10)$$

where

$$\overline{v_{p10}^2} = \overline{v_{p10x}^2} + \overline{v_{p10y}^2} + \overline{v_{p10z}^2}, \qquad \overline{\left(\frac{\partial v_{p_{10}}}{\partial x}\right)^2} = \overline{\left(\frac{\partial v_{p_{10x}}}{\partial x}\right)^2} + \overline{\left(\frac{\partial v_{p_{10y}}}{\partial x}\right)^2} + \overline{\left(\frac{\partial v_{p_{10z}}}{\partial x}\right)^2}$$

and $\overline{v_{p_{10}} \cdot v_{p_{20}}} = \overline{v_{p_{10x}} v_{p_{20x}}} + \overline{v_{p_{10y}} v_{p_{20y}}} + \overline{v_{p_{10z}} v_{p_{20z}}}$.

The variance of particle relative velocities, Eq. 10, is expressed by the turbulent intensity of each particle, the particle velocity correlation, and the variance of particle velocity gradients and their correlation. Since these physical variables are turbulent characteristics of particles which are very difficult to measure, they should be expressed instead by the turbulent characteristics of a fluid verified by means of the Lagrangian-type equation of particle motion.

Behavior of Particle and Fluid vs. Turbulence

Hinze (1975) derived Lagrangian-type equation of particle motion under the assumptions that the turbulent flow field is isotropic and homogeneous, that the particle is spherical, and so small that its motion relative to the fluid obeys Stokes' law, and that the surrounding area during the motion of the particle is formed by the same fluid. The i component of the equation is as follows:

$$\frac{dv_{pi}}{dt} + a_j v_{pi} = a_j v_{fi} + b \frac{\partial v_{fi}}{\partial t} + \frac{18}{2\rho_p + \rho_f} \sqrt{\frac{\rho\mu}{\pi}} \int_{t_0}^{t} \frac{\frac{dv_{fi}}{dt'} - \frac{dv_{pi}}{dt'}}{\sqrt{t - t'}} dt' \quad (11)$$

where a_j is the reciprocal relaxation time defined as $a_j = 36\mu / C_c(2\rho_p + \rho_f) D_{pj}^2$ when particle motion obeys Stokes' law as corrected by Cunningham's slip factor C_c and b is the buoyancy coefficient, defined as $b = 3\rho_f/(2\rho_p + \rho_f)$. Cunningham's slip factor is $C_c = 1 + [2.46 + 0.82\exp(-0.44 D_{pj}/\lambda)]\lambda/D_{pj}$.

Hughes and Gilliland (1952) and Hinze (1975) indicated that the third term on the righthand side of Eq. 11, the Basset term, becomes important only when the density of the fluid becomes comparable to or higher than that of the particles. Therefore, in this study, the Basset term has been neglected and Eq. 11 simplified to the following.

$$\frac{dv_{pi}}{dt} + a_j v_{pi} = a_j v_{fi} + b \frac{\partial v_{fi}}{\partial t}. \quad (12)$$

The turbulent velocities of fluid and particle are then expressed by using the Fourier integral as follows:

$$v_{fi} = 2\pi \int_0^\infty [\alpha_i(M)\cos(2\pi Mt) + \beta_i(M)\sin(2\pi Mt)]dM.$$
$$v_{p10i} = 2\pi \int_0^\infty [\gamma_{1i}(M)\cos(2\pi Mt) + \delta_{1i}(M)\sin(2\pi Mt)]dM. \quad (13)$$
$$v_{p20i} = 2\pi \int_0^\infty [\gamma_{2i}(M)\cos(2\pi Mt) + \delta_{2i}(M)\sin(2\pi Mt)]dM.$$

Substitution of Eq. 13 into Eq. 12 yields

$$\gamma_{1i} = \left[1 + \frac{\omega^2(b-1)}{a_1^2 + \omega^2}\right]\alpha_i + \frac{a_1\omega(b-1)\beta_i}{a_1^2 + \omega^2},$$
$$\gamma_{2i} = \left[1 + \frac{\omega^2(b-1)}{a_2^2 + \omega^2}\right]\alpha_i + \frac{a_2\omega(b-1)\beta_i}{a_2^2 + \omega^2}$$
$$\delta_{1i} = -\left[\frac{a_1\omega(b-1)}{a_1^2 + \omega^2}\right]\alpha_i + \left[1 + \frac{\omega^2(b-1)}{a_1^2 + \omega^2}\right]\beta_i,$$
$$\delta_{2i} = -\left[\frac{a_2\omega(b-1)}{a_2^2 + \omega^2}\right]\alpha_i + \left[1 + \frac{\omega^2(b-1)}{a_2^2 + \omega^2}\right]\beta_i \quad (14)$$

where $\omega = 2\pi M$.

Equation 14 gives a relation between the velocities of particle and fluid which satisfies Eq. 12.

Now $\overline{v_{p10i} v_{p20i}}$ in Eq. 10 is derived as follows.

$$\overline{v_{p10i}(\tau)\, v_{p20i}(\tau - t)} = \frac{1}{2T - t} \int_{-T_1}^{T} v_{p10i}(\tau) v_{p20i}(\tau - t) d\tau \quad (15)$$

where $T_1 > T$.

Substitution of Eq. 13 into Eq. 15 and its rearrangement yields

$$\overline{v_{p10i}(\tau)\, v_{p20i}(\tau)} = \pi^2 \int_0^\infty \frac{\gamma_{1i}(M)\gamma_{2i}(M) + \delta_{1i}(M)\delta_{2i}(M)}{T} dM = \int_0^\infty E_{pL1,2i}(M) dM \quad (16)$$

where $t = 0$

Similarly the turbulent intensity of the fluid is represented by

$$v_{f0i}^2(\tau) = \pi^2 \int_0^\infty \frac{\alpha_i^2(M) + \beta_i^2(M)}{T} dM$$

$$= \int_0^\infty E_{fL_i}(M)dM. \quad (17)$$

Combining Eq. 16 with Eq. 17 gives

$$E_{pL_{1,2i}}(M) = \frac{\gamma_{1i}(M)\gamma_{2i}(M) + \delta_{1i}(M)\delta_{2i}(M)}{\alpha_1^2(M) + \beta_2^2(M)} E_{fL_i}(M). \quad (18)$$

Using the experimental relation $R_{fL_i}(\tau) = \exp(-\tau/T_{fL_i})$, Hinze (1975) derived the following equation.

$$E_{fL_i}(M) = 4v_{f0i}^2 \frac{T_{fL_i}}{1 + \omega^2 T_{fL_i}^2}. \quad (19)$$

Substitution of Eq. 18, expressed in a new form by using Eqs. 14 and 19 into Eq. 16 gives:

$$\overline{v_{p10i}(\tau)v_{p20i}(\tau)} = \frac{2}{\pi}\overline{v_{f0i}^2}\, T_{fL}$$

$$\int_0^\infty \frac{(a_1a_2 + \omega^2)(a_1a_2 + \omega^2 b) + (a_1 - a_2)^2\omega^2 b}{(a_1^2 + \omega^2)(a_2^2 + \omega^2)(1 + \omega^2 T_{fL}^2)} d\omega = B\,\overline{v_{f0i}^2}$$

where B is defined as

$$B = \frac{C}{(a_1 + a_2)(1 - a_1^2T_{fL}^2)(1 - a_2^2T_{fL}^2)}$$

and C represents

$$C = a_1a_2T_{fL}[2 - (a_1 + a_2)T_{fL} - (a_1^2 + a_2^2)T_{fL}^2 + a_1a_2(a_1 + a_2)T_{fL}^3] + (a_1 - a_2)^2T_{fL}[1 - (a_1 + a_2)T_{fL} + a_1a_2T_{fL}^2]b + [(a_1 + a_2) - (a_1^2 + a_2^2)T_{fL} - a_1a_2(a_1 + a_2)T_{fL}^2 + 2a_1^2a_2^2T_{fL}^3]b^2. \quad (20)$$

where the flow field is assumed to be isotropic and T_{fL_i} equals T_{fL}. Hinze obtained the following relation between the turbulent intensities of fluid and particle:

$$\overline{v_{pk0i}^2} = \left(\frac{a_kT_{fL} + b^2}{a_kT_{fL} + 1}\right)\overline{v_{f0i}^2} = A_k\,\overline{v_{f0i}^2}. \quad (21)$$

The following equation holds when the turbulent velocity field is homogeneous.

$$\overline{\frac{\partial v_{pi}}{\partial x_k}\frac{\partial v_{pj}}{\partial x_1}} = \frac{\partial}{\partial x_k}\frac{\partial}{\partial x_1}\overline{(v_{pi}v_{pj})}. \quad (22)$$

Thus each term of Eq. 10 is able to be transformed by using Eq. 22 as follows:

$$\overline{\left(\frac{\partial v_{p10}}{\partial x}\right)^2} = \frac{\partial^2}{\partial x^2}\overline{v_{p10}^2};\quad \overline{\frac{\partial v_{p10}}{\partial x}\cdot\frac{\partial v_{p20}}{\partial x}} = \frac{\partial^2}{\partial x^2}\overline{v_{p10}\cdot v_{p20}}. \quad (23)$$

Substitution of Eqs. 20–22 into Eq. 10 yields:

$$\overline{w^2} = (A_1 - 2B + A_2)\overline{v_{fo}^2} + (A_1r_{p1x}^2 + 2Br_{p1x}r_{p2x} + A_2r_{p2x}^2)\overline{\left(\frac{\partial v_{fo}}{\partial x}\right)^2} + (A_1r_{p1y}^2 + 2Br_{p1y}r_{p2y} + A_2r_{p2y}^2)\overline{\left(\frac{\partial v_{fo}}{\partial y}\right)^2} + (A_1r_{p1z}^2 + 2Br_{p1z}r_{p2z} + A_2r_{p2z}^2)\overline{\left(\frac{\partial v_{fo}}{\partial z}\right)^2}. \quad (24)$$

When the turbulent flow field is isotropic,

$$\overline{\left(\frac{\partial v_{fo}}{\partial x}\right)^2} = \overline{\left(\frac{\partial v_{fo}}{\partial y}\right)^2} = \overline{\left(\frac{\partial v_{fo}}{\partial z}\right)^2} = \frac{1}{3}\epsilon/\nu. \quad (25)$$

Substitution of Eq. 25 into Eq. 24 gives:

$$w^2 = (A_1 - 2B + A_2)\overline{v_{fo}^2} + (A_1r_{p1}^2 + 2Br_{p1}r_{p2} + A_2r_{p2}^2)\left(\frac{1}{3}\frac{\epsilon}{\nu}\right) \quad (26)$$

and

$$A_1 - 2B + A_2 = \frac{(a_1 - a_2)^2(1 - b)^2T_{fL}}{(a_1 + a_2)(1 + a_1T_{fL})(1 + a_2T_{fL})}. \quad (27)$$

Substitution of Eq. 26 into Eq. 6 gives

$$N_{12} = \left(\frac{8\pi}{3}\right)^{1/2}\eta(r_{p1} + r_{p2})^2n_1n_2\left[(A_1 - 2B + A_2)v_f^2 + (A_1r_{p1}^2 + 2Br_{p1}r_{p2} + A_2r_{p2}^2)\frac{\epsilon}{3\nu}\right]^{1/2} \quad (28)$$

where $v_f^2 = v_{f0}^2$ because the flow field is homogeneous.

We are interested primarily in collisions due to the turbulent motion of particles. Accordingly, we did not consider here the effect of distortion of the flow due to the presence of a particle (collision efficiency). So the coagulation coefficient, which is necessary to evaluate the coagulation process, is defined as $K_t = N_{12}/n_1n_2\eta$, Eq. 28 is then expressed as

$$K_t = \left(\frac{8\pi}{3}\right)^{1/2}(r_{p1} + r_{p2})^2\left[(A_1 - 2B + A_2)v_f^2 + (A_1r_{p1}^2 + 2Br_{p1}r_{p2} + A_2r_{p2}^2)\frac{\epsilon}{3\nu}\right]^{1/2}, \quad (29)$$

which is our final equation.

RESULTS AND DISCUSSION

Each particle moves relative to the surrounding fluid due to the inertia. Neighboring particles of unequal size have different velocities because of their different inertias thus causing collisions. This type of collision is defined by Saffman and Turner (1956) as "collision due to the motion relative to the fluid." In addition, spatial variation of turbulent motion gives neighboring particles different velocities which also cause collisions. This type of collision is called "collision due to the motion of the particles with the fluid."

The collision of two particles (Figure 1) is attributable to the effects of both the particle motion relative to the fluid and the turbulent motion imparted by the fluid. The first term of the righthand side of Eq. 29, $(A_1 - 2B + A_2)\overline{v_{f0}^2}$, represents the former effect, and the second term, $(A_1r_{p1}^2 + 2Br_{p1}r_{p2} + A_1r_{p2}^2)\epsilon/3\nu$, the latter effect. It can be seen from Eqs. 20, 21 and 27, that A_1, A_2, and B are the coefficients expressing the measure of velocity lag between particle and fluid, and are the functions of particle relaxation time, integral time scale, and intensity of turbulent flow. As integral time scale in isotropic turbulence is equal to $0.265\,\overline{v_f^2}/\epsilon$, A_1, A_2, and B are also functions of the dissipation rate of turbulence energy. If the particles follow the fluid flow completely or if the particle relaxation times become zero, A_1, A_2 and B become equal to unity, and K_t is reduced to

$$K_t = 1.67(r_{p1} + r_{p2})^3\left(\frac{\epsilon}{\nu}\right)^{1/2}. \quad (30)$$

Equation 30 is almost the same as the equation derived by Saffman and Turner (1956).

If particles do not follow the fluid flow, that is, if the particle relaxation times or the particle inertias are infinitely large, A_1, A_2 and B become zero and K_t becomes zero. In other words, the particles do not collide due to turbulence. Thus, Eq. 28 overcomes the first contradiction in Eq. 2 as pointed out above.

If $r_{p1} = r_{p2}$, $A_1 = A_2 = B$. That is, when particles with equal radii collide, the term $(A_1 - 2B + A_2)\overline{v_{f0}^2}$ becomes zero. Accordingly, the collisions are then due primarily to the effect of the spatial distribution of the turbulent components of the particles. Since the velocity lag between particle and fluid has been taken into consideration, A_1, A_2 and B approach zero with increasing particle relaxation time and become zero as the time reaches infinity. Therefore, Eq. 28 overcomes the second contradiction in Eq. 2.

Pismen and Nir (1978) had calculated the particle velocity correlation and the diffusivity in a homogeneous turbulent field by using the "independence approximation" proposed by Lundgren and Pointin (1976) and Weinstock (1976), which is equivalent to Corrsin's conjecture (1959). These calculations of Pismen and Nir (1978) are important in understanding the turbulent mechanism of particles. However, there are some limitations. For example, their results show that the nondimensional particle diffusivity $\overline{D}$ has its maximum at the reciprocal particle relaxation time $a_j = 0$. If the particle collision rate is then calculated at this point, the result indicates that those particles which have no turbulent random motions have the highest collision frequency. This is obviously incorrect and leads to the conclusion that the "independence approximation" (Corrsin's conjecture) should not, therefore, be applied to the calculation of small values of a_j.

In this paper the particle velocity correlation is calculated by using the empirical formula for the Lagrangian fluid velocity correlation $R_{fL} = \exp(-\tau/T_{fL})$. Examples of the calculated coagulation coefficients using different equations are presented in Figures 2 and 3. At the points where $r_{p_1} = r_{p_2}$, the line for Eq. 29 shows sharp valleys. This is because at this point the motion relative to the air is now zero and collisions now occur due only to the weaker factor of the spatial distribution of turbulence. When r_{p_1} is 0.5 μm and r_{p_2} is less than 0.3 μm (Figure 2), the calculated coagulation coefficient of the present study is about five times larger than that calculated from Eq. 2. The coagulation coefficient from Eq. 2 increases with increasing r_{p_2} (from $r_{p_2} = r_{p_1}$) and finally tends to infinity, whereas our results decrease at some large particle radius and finally tend to approach zero due to large particle inertia. These differences between Eqs. 29 and 2 are mainly due to neglect of the acceleration term of the equation of particle motion in the derivation of Eq. 2. The collision coefficient also decreases with decreasing particle radii. When $r_{p_1} = 0.05$ μm and r_{p_2} drops below 0.1 μm (Figure 3) the coagulation coefficient due to turbulence, where now $\epsilon = 10^6$ (cm^2/g^2) and $\overline{v_f^2} = 10^4$(cm^2/s^2), becomes smaller than that due to Brownian movement. Therefore, in this region of very small particles, the primary cause of collisions becomes Brownian movement.

The value of K_t for equal size particles is shown in Figure 4. The K_t calculated by Eq. 29 decreases with increasing ρ_p because the particles find it more difficult to follow the turbulent fluid flow as particle inertia increase. However, if calculated by Eq. 2 K_t

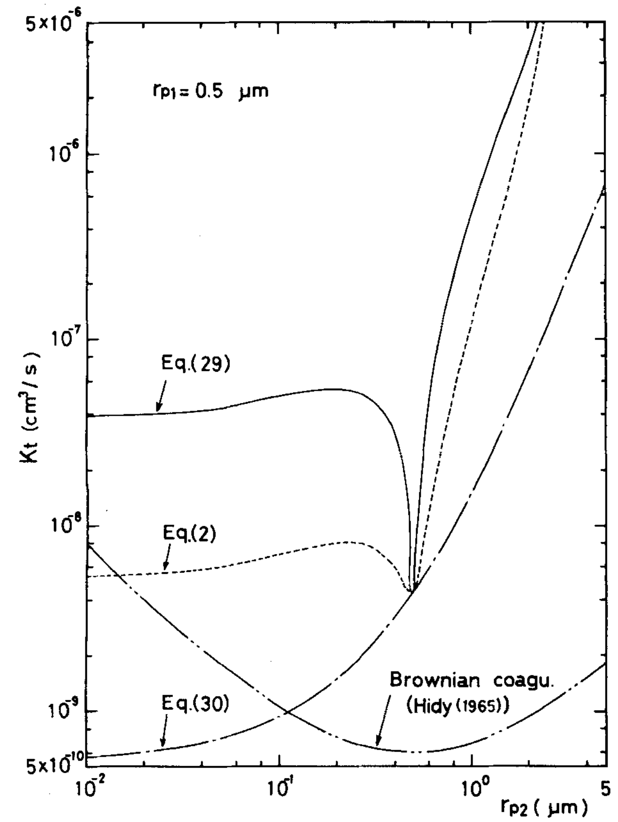

Figure 2. Coagulation coefficients of small particles in air stream (ϵ = 10^6cm^2/s^3, $\overline{v_f^2}$ = 10^4cm^2/s^2, ρ_p = 2.0g/cm^3).

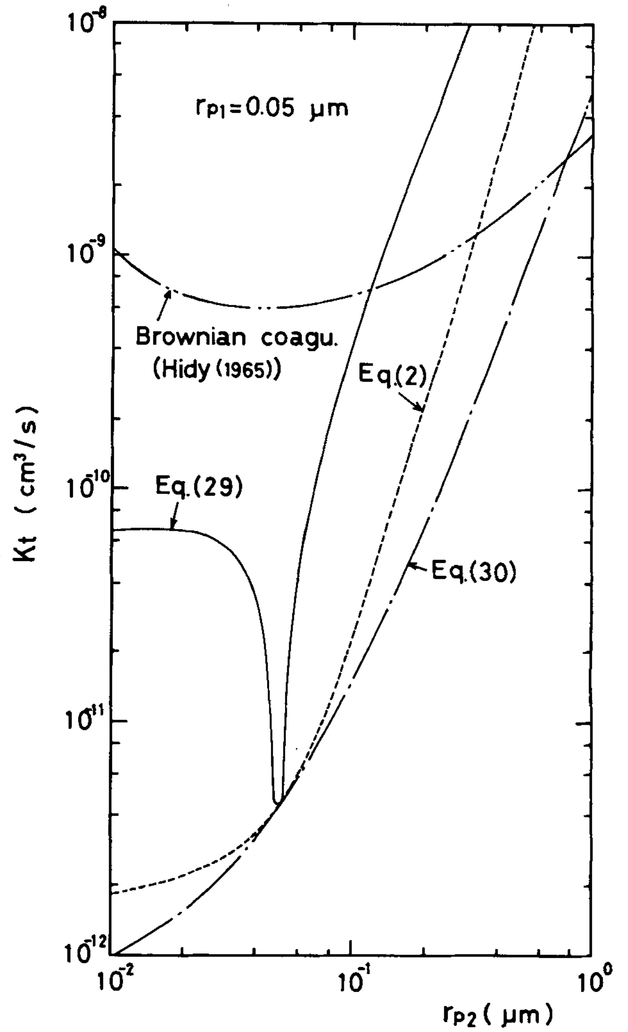

Figure 3. Coagulation coefficients of small particles in air stream (ϵ = 10^6cm^2/s^3, $\overline{v_f^2}$ = 10_4cm^2/s^2, ρ_p = 2.0 g/cm^3).

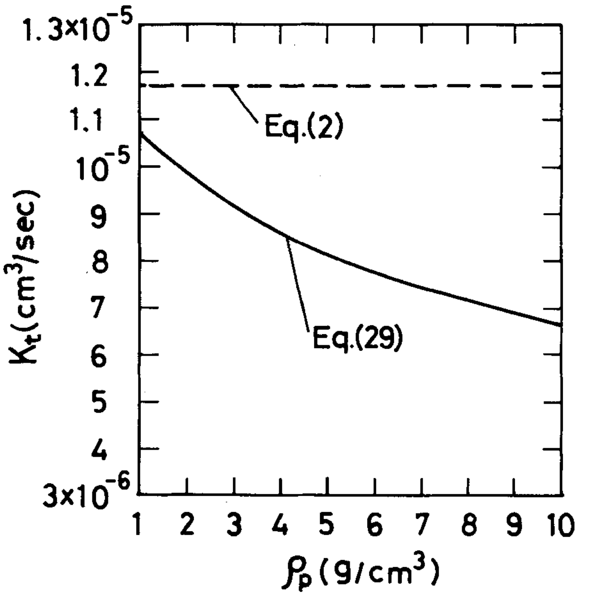

Figure 4. Coagulation coefficients in air stream (ϵ = 2.5 × 10^6cm^2/s^3, $\overline{v_f^2}$ = 2 × 10^4cm^2/s^2, ρ_p = 2.0g/cm^3, $r_{p_1} = r_{p_2}$ = 6μm).

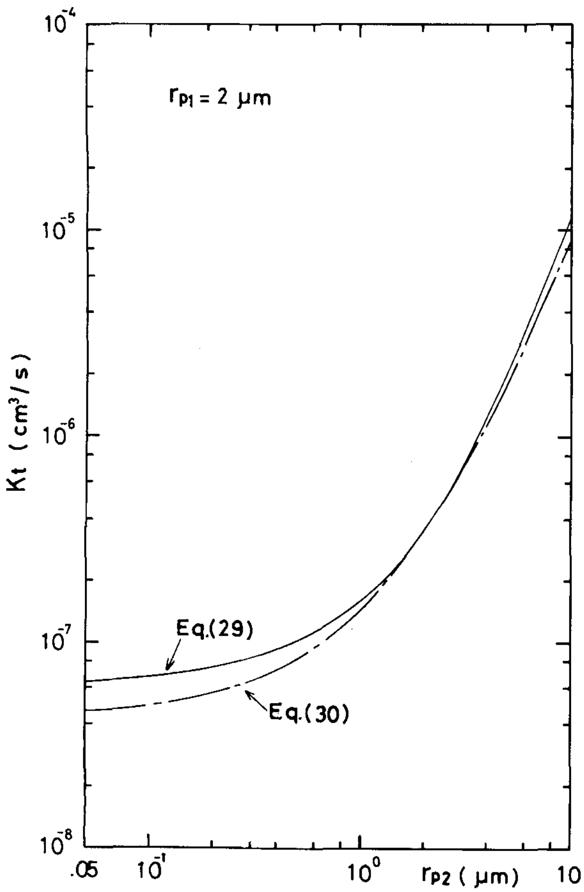

Figure 5. Coagulation coefficients of small particles in water stream (ϵ = $10^4 cm^2/s^3$, $\overline{v_f^2}$ = $10^2 cm^2/s^2$, ρ_p = 2.0 g/cm³, ρ_f = 1.0 g/cm³).

would remain constant in spite of the increase of ρ_p. This is contrary to what actually happens in a suspension.

Figure 5 shows the comparison of the coagulation coefficients of Eqs. 29 and 30 for the case of a water stream. This figure indicates that here the collision due to the spatial variation of turbulence is the predominant factor in the coagulation process for the reason that the particle inertia in a water stream is much smaller than that in an air stream. However, the K_t from Eq. 29 for the different sized particles is larger than that from Eq. 30 by about 20% because there is still some motion relative to the water.

ACKNOWLEDGMENT

The author is indebted to James Hansen and Eder Hansen for many helpful suggestions and advice in writing this paper in English.

NOTATION

A_k = value defined by Eq. 21
a_j = reciprocal of particle relaxation time, $36\mu/C_c(2\rho_p + \rho_f)D_{pj}^2[s^{-1}]$
B = value defined by Eq. 20
C_c = Cunningham's correction factor, $1 + [2.46 + 0.82 \exp(-0.44D_{pj}/\lambda)]\lambda/D_{pj}$
D_{pj} = diameter of j particle [μm]
E_{fL}, E_{pL} = fluid and particle Lagrangian energy spectrums $[cm^2 \cdot s^{-1}]$
K_t = coagulation coefficient [cm³/s]
M = frequency $[s^{-1}]$
N_{12} = collision rate $[cm^{-3} \cdot s^{-1}]$
n_1, n_2 = number concentrations of particles of diameter r_{p_1} and r_{p_2} $[cm^{-3}]$
R_{fL} = Lagrangian fluid correlation coefficient
r_{pi} = radius of i particle or position vector $(r_{pix}, r_{piy}, r_{piz})$ [μm]
T_{fL} = Lagrangian fluid integral time scale [s]
v_f, v_p = fluid and particle turbulent velocity vectors (v_{fx}, v_{fy}, v_{fz}), (v_{px}, v_{py}, v_{pz}) [cm/s]
w = relative velocity vector of two colliding particles, (w_x, w_y, w_z) [cm/s]

Greek Letters

ϵ = average energy dissipation rate [cm²/s³]
η = collision efficiency
λ = mean free path [μm]
μ, ν = fluid viscosity and fluid kinetic viscosity [g/cm · s] [cm²/s]
ρ_f, ρ_p = fluid and particle densities [g/cm³]
τ_{pi} = relaxation time of i particle [s]
——— = denotes time averaged quantity [s]

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Manuscript received April 29, 1982; revision received June 9, and accepted July 11, 1983.